WORLD-VOLUMES AND STRING TARGET SPACES

Michael B. Green[1]

DAMTP, Silver Street,
Cambridge CB3 9EW, UK

ABSTRACT

String duality suggests a fascinating juxtoposition of world-volume and target-space dynamics. This is particularly apparent in the D-brane description of stringy solitons that forms a major focus of this article[2] (which is *not* intended to be a comprehensive review of this extensive subject). The article is divided into four sections:

- The oligarchy of string world-sheets

- p-branes and world-volumes

- World-sheets for world-volumes

- Boundary states, D-branes and space-time supersymmetry

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[1]M.B.Green@damtp.cam.ac.uk
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1 The oligarchy of string world-sheets

According to current lore all known superstring perturbation expansions may be viewed as different approximations to a single underlying theory. Viewed from the standpoint of any given string theory the fundamental strings of other theories are BPS solitons that are not apparent as particle states in string perturbation theory. In addition there are other solitonic solutions with p-dimensional spatial extension which play a vital rôle in the duality symmetries of the theory. All of these solitons have the property that they preserve some fraction of space-time supersymmetry so that they are analogous to the BPS monopoles in the Higgs–Yang–Mills system. The interrelations between different perturbation expansions [1, 2] (and references therein) appear to be profound and should indicate the path towards a more fundamental way of expressing the theory in which no particular perturbative approximation has preferred status.

However, strings ($p = 1$) do have an exalted position among this panoply of solitons. Only those theories in which the fundamental states are strings possess well-defined (albeit non-convergent) world-volume and target-space perturbation expansions. Thus, although first-quantized point-particle ($p = 0$) quantum mechanics is well-defined, a theory including gravity cannot be second-quantized in perturbation approximation. Moreover, p-brane quantum mechanics with $p > 1$ is ill-defined – the world-volume theories for such p-branes are non-renormalizable quantum theories and can only be interpreted as effective theories, suitable for describing long wavelength behaviour.

But, following recent developments it has become plausible that the $(p + 1)$-dimensional world-volume field theories of p-branes should be interpreted as effective low-energy approximations to an underlying string theory. This is implicit in the D-brane description of p-branes carrying the charges of the Ramond–Ramond ($R \otimes R$) sector [3, 4] (and a multitude of other papers) and plausibly (but more speculatively) of the solitonic heterotic string and fivebrane [5] which carry Neveu–Schwarz - Neveu–Schwarz ($NS \otimes NS$) charges.

This represents the implementation of the idea that the string world-sheet of one string theory (the 'effective' theory) might be interpreted as an effective two-dimensional target-space of another string theory (the 'underlying' theory) [6]. The world-sheet co-ordinates of the effective theory are world-sheet fields of the underlying theory while the world-sheet fields of the effective theory are target-space fields of the underlying theory. The string tension of the effective theory (which is the effective world-sheet loop counting parameter) is then determined in terms of the target-space closed-string coupling constant, g, of the underlying theory. If the underlying string theory is a closed-string theory the effective string tension behaves as $1/g^2$ while if it is an open-string theory the effective string tension behaves as $1/g$. In this way there is a kind of 'world-sheet democracy' among string theories.

It is natural to extend this idea to the description of p-branes with the $(p + 1)$-

1

dimensional world-volume field theory replaced by a corresponding string theory. Thus, world-volume dynamics should be well-defined in terms of the underlying string theory. In this sense there is a 'world-sheet oligarchy' controlling the dynamics of p-branes. Indeed, in the D-brane description of $R \otimes R$ p-branes [3, 4] the underlying theory is an open superstring theory and (more speculatively) in the cases of the heterotic string and five-brane the underlying theory is type IIA closed superstring theory [5] (in which the world-volume fields are twisted closed strings pinned to an orbifold point).

This leaves open the question of the relation of string theory to eleven-dimensional supergravity – or rather to M-theory [7], a Mythical eleven-dimensional quantum mechanical theory from which all known string perturbation theories can be obtained by compactification to lower dimensions. M-theory should possess a two-brane soliton as well as its magnetic five-brane partner, which arise as classical solutions of eleven-dimensional supergravity – the 'classical limit' of M-theory. These solitons reduce to a variety of both $NS \otimes NS$ and $R \otimes R$ p-branes in lower dimensions but it is currently a mystery as to how the eleven-dimensional solitons may be expressed in terms of a fundamental quantum theory of strings. The possibility that a twelve-dimensional supersymmetric theory with $(10, 2)$ signature – 'F-theory' – should play a rôle in these ideas has been illuminated in a recent interesting paper [8] that makes use of the properties of the string theory with local $N = 2$ supersymmetry. It may be that M 'theory' and F 'theory' can be understood as some combination of the ideas in [6] and [8].

Another issue that is related to p-brane dynamics is the possible existence of a new energy scale in string theory beyond the string scale [9], which might be identified with the scale for exciting higher soliton modes. This may also be connected to the presence of point-like fixed-angle scattering at high energy in string perturbation theory in the presence of Dirichlet boundaries [10].

The next two sections of this article will review certain features of p-branes, D-instantons [11, 12] and D-branes in order to illustrate these points. In section 4 space-time supersymmetry of D-branes and some of their interactions will be described using methods that were originally applied to the case of purely Dirichlet boundary conditions $(p = -1)$ [13, 12]. Actually, the arguments apply to '$(p + 1)$-instantons' [14]. These are related to p-branes by a double Wick rotation so that the world-volume is euclidean and time is identified with one of the directions transverse to the $(p + 1)$-dimensional world-volume.

2 p-branes and world-volumes

In the original type IIA and IIB superstring theories the fundamental string states carry charges that couple to the massless antisymmetric tensor potential, $B_{\mu\nu}^N$, of the $NS \otimes NS$ sector. The string-like soliton in the $NS \otimes NS$ sector of either theory is a source for the 'electric' B^N charge while its 'magnetic' partner is a five-brane solution carrying the

magnetic B^N charge. [In general an 'electric' p-brane charge is the integral of $*(F_{p+2})$ on the $(8-p)$-sphere that bounds the space outside the charged object in ten dimensions while its 'magnetic' partner is carried by a $(6-p)$-brane which has a magnetic charge given by the integral of F_{p+2} on the $(p+2)$-sphere surrounding it.]

In addition to the two-form potential of the $\text{NS} \otimes \text{NS}$ sector there are several other massless $(p+1)$-form potentials (with $(p+2)$-form field strengths, F_{p+2}) that arise as fundamental string states of the $\text{R} \otimes \text{R}$ sector. These comprise: a (which is the $\text{R} \otimes \text{R}$ pseudoscalar, or zero-form), $B^R_{\mu\nu}$ and $A_{\mu\nu\rho\tau}$ in the type IIB theory and A_μ and $A_{\mu\nu\rho}$ in the type IIA theory. Thus, there are potentials of rank $0, 1, 2, 3, 4$ in the $\text{R} \otimes \text{R}$ sectors of the type II theories. Although there are conserved charges associated with these potentials there are no fundamental string states that carry these charges. This is reflected by the fact that the potentials do not couple minimally to the fundamental string world-sheet. However, p-brane solitons could carry the electric or magnetic $\text{R} \otimes \text{R}$ charges with p taking any of its possible values from $p = -1$ to $p = 9$. Such solutions of the type II supergravity theories have indeed been found for all values of p in its range, $-1 \leq p \leq 9$. The zero-brane (black hole), two-brane, four-brane, six-brane and eight-brane solutions of the type IIA theory and the one-brane (string), self-dual three-brane, five-brane and seven-brane solitons of type IIB string are standard solitons (many of these are reviewed in [15]).

There is actually an infinite number of 'dyonic' $\text{R} \otimes \text{R}$ one-branes in the type IIB theory carrying (coprime) integer values of the charges that couple to the antisymmetric tensors in the $\text{NS} \otimes \text{NS}$ and $\text{R} \otimes \text{R}$ sectors [16]. Likewise, there is an infinite set of five-branes carrying the magnetic charges. The case $p = 9$ is special since the accompanying field strength vanishes identically, and the 10-form potential itself is connected with the presence of chiral anomalies in type I theories with any gauge group other than $SO(32)$. The $p = 8$ soliton couples to a cosmological constant in the type IIA theory [3, 17, 18] and is a solution of 'massive' type IIA supergravity [19].

The world-volume field theory that defines the classical dynamics of each of these p-branes is a $(p+1)$-dimensional field theory with extended supersymmetry. For $p > 2$ this necessarily involves world-sheet fields with spins greater than $1/2$ – for example, a brane of the $\text{R} \otimes \text{R}$ sector is described by a world-volume theory that is a supersymmetric abelian gauge theory [20, 21] in which one of the fields is the spin-1 photon. This world-volume gauge potential emerges from zero modes of the ten-dimensional theory fields in the background of a brane, whereas in the D-brane picture it is identified with a ground state of the open superstring.

The case of the -1-brane deserves special attention since it is an instanton rather than a solitonic particle. Instead of carrying a charge, such a solution describes a transition which leads to the non-conservation of the charge to which it couples. This is the Noether charge associated with the translation symmetry of the $\text{R} \otimes \text{R}$ scalar potential, a, in the classical theory. Unlike other $\text{R} \otimes \text{R}$ charges this is a global charge and its violation is to be expected in string theory. In fact, the instanton solution [22] has the string-frame

interpretation of a space-time Einstein–Rosen wormhole [23] connecting two asymptotically flat regions. Particles carrying the charge can fall through the neck connecting one region to the other which leads to its non-conservation.

The fact that these solutions preserve some fraction of the maximal original supersymmetries means that they have much more tractable quantum corrections than generic, non-supersymmetric, solutions and it follows that certain statements about the strong coupling limit are reliable. The prototype for this is the Montonen–Olive conjecture [24, 25, 26] relating weak and strong coupling limits of $N = 4$ supersymmetric Yang–Mills theory generalized to a conjectured $SL(2, Z)$ symmetry of the theory [27]. The field theories to which this conjecture and its generalizations apply may be viewed as low energy limits of superstring theories and this suggests a generalization of the conjecture to the string setting. There is by now much evidence for these stringy non-perturbative dualities.

3   World-sheets for world-volumes

D-branes and D-instantons
The p-brane solitons carrying charges in the $R \otimes R$ sector of the low energy supersymmetric field theory have an intrinsically stringy manifestation. These stringy versions of branes, known collectively as D-branes [28, 29, 3] (or D-instantons when $p = -1$ [11]), are associated with *open* superstring configurations.

The essential point is that a solitonic solution of perturbative field theory has a mass (more properly, a mass per unit volume) that becomes infinite in the limit of zero coupling, $g \to 0$ – typically the mass behaves as $1/g^2$ or (in the case of the solitons of the $R \otimes R$ sector) $1/g$. Perturbation theory in the presence of a soliton is not manifestly invariant under ten-dimensional translation invariance. So if the brane world-volume is taken to be a flat lorentzian sheet in $p + 1$ dimensions translation invariance is spontaneously broken in the $(9 - p)$ transverse dimensions and is only restored after integration over translational zero modes. There are interesting open superstring configurations which describe fluctuations around just this kind of background. These are open superstrings with their end-points fixed in the world-volume of the brane – in other words in the directions labelled by $i = p + 1, \ldots, 9$ the coordinates satisfy $\partial_t X^i = 0$ (where t denotes the derivative tangential to the boundary). In the other directions, X^α with $\alpha = 0, \ldots, p$, the end-points satisfy the usual open-string Neumann boundary conditions, $\partial_n X^\alpha = 0$ (where n denotes the normal derivative at a world-sheet boundary). These configurations preserve half of the space-time supersymmetries – they are stringy analogues of BPS states. So we see that whereas the standard perturbative states of type II superstring theories are closed-string configurations, in the presence of p-brane solitons there are additional open-string states in the theory with end-points moving in the brane. These open-string configurations owe their existence to the presence of the brane to which they

are tethered.

The coupling of these open superstrings to the brane give rise to fluctuations that define the brane dynamics. Thus, a pair of string end-points may be created in the world-volume of the brane at some time and annihilate at a subsequent time. In between these times the end-points trace out a closed curve in the world-volume. The intermediate open-string histories define a world-sheet that is a disk bounded by this curve. This means that the boundary conditions on the disk are Neumann in $p + 1$ dimensions and Dirichlet in $9 - p$ dimensions. The functional integral over the disk gives the leading contribution to the ground-state energy of the D-brane that is of order $c/g \sim ce^{-<\phi>}$ where c is a known constant [11]. The factor of $1/g$, which here arises from the fact that the Euler character of a disk is -1, is characteristic of the energy of solitons in the $R \otimes R$ sector.

Independent fluctuations of the brane can arise in disconnected regions of the world-volume, described by independent disk world-sheets. The case of the D-instanton is special since the world-sheet boundaries are fixed at a position y^μ ($\mu = 0, \ldots, 9$) [11] which is a constant. Since the boundaries of the disks are all at the same space-time point momentum is conserved after integration over the space-time position of the instanton. With $p > -1$ momentum is not conserved locally unless account is taken of the excitation of long wavelength modes of the brane. These modes carry the momentum between the disconnected regions spanned by the separate disks.

The full string partition function involves a sum over all possible numbers of such fluctuations and this gives rise to an exponential of the individual disk diagram in the functional integral. In addition to these iterations of disconnected disk diagrams there are iterations of diagrams with more complicated topology that also contribute to the dynamics of a single D-brane. The simplest of these is the diagram in which the boundaries of two disks form the ends of a cylindrical world-sheet. This describes a higher-order vacuum effect in which a closed string is exchanged between the two boundaries. Multiple iterations of this diagram must be included in the functional integral and these also exponentiate. The general diagram contributing to a single brane consists of the exponential of connected world-sheets with arbitrary numbers of boundaries and handles.

For example, in the background of a single D-instanton the sum over all diagrams contributing to the vacuum functional is given by

$$Z = \int d^{10}y \, e^{S^{(1)}}. \tag{1}$$

In this expression the single D-instanton action, $S^{(1)}$, is given by

$$S^{(1)} = \sum_{r=0}^{\infty} g^{r-2} f_r(y), \tag{2}$$

where f_r is the functional integral over *connected* world-sheets with r boundaries (and there is also a sum over handles that has been suppressed) [12]. The background fields

are required to satisfy the conditions that ensure conformal invariance and the fluctuations of these fields determine the perturbative vertex operators in the usual manner. In the absence of fundamental external closed-string states the $r = 0$ term (the spherical world-sheet) vanishes by the usual scaling argument and the terms with $r > 1$ vanish by supersymmetry, leaving simply the disk diagram of order $1/g$ in the exponent of (1). The instanton therefore generates non-perturbative contributions to the scattering amplitudes of the form $e^{c(ia_0 - 1/g)}$ (where a_0 is the constant value of the $R \otimes R$ scalar field that is a parameter of the instanton solution). The occurrence of such effects in string theory was anticipated by Shenker [30] by considering the rate of divergence of string perturbation theory and is strikingly different from soliton and instanton effects in standard field theories such as QCD which are typically of order e^{-c/g^2}.

The generalization of (1) to the situation in which there are arbitrarily many BPS (and no anti-BPS) D-instantons is straightforward [12].

Scattering in the presence of D-instantons and D-branes
Fundamental closed-string states scatter from a D-brane by interacting with the open-string fluctuations. The amplitude is given in leading approximation by attaching closed-string vertex operators for the fundamental string states to the disk world-sheet. If the brane is fixed in space at a constant transverse position y^i $(i = p+1, \ldots, 9)$ momentum is not conserved in the process but once the zero modes of the D-brane are taken into account (which include the overall translation mode) momentum conservation will be restored. In general, there are bosonic zero modes associated with local transverse fluctuations of the brane so that the position of the brane is the field, $Y^i(X^\alpha)$. According to [3] and [4] infinitesimal fluctuations are induced by coupling the open-string massless vector potential to the boundary of the disk. This vertex operator has the form $\oint d\sigma A_i(X^\alpha) \partial_n X^i$, where $A_i(X^\alpha) = Y^i(X^\alpha) - y^i$ are the components of the vector potential transverse to the brane at the point X^α. The brane can be given a uniform velocity by choosing A^i to be linear in X^0 [31, 32]. The other components of the vector potential, $A_\alpha(X^\beta)$, comprise a world-volume vector field. In addition, the fermionic zero modes associated with the broken supersymmetries are excited by coupling to the massless open-string fermion vertex.

Scattering amplitudes of fundamental closed-string states obtained from (1) by considering small fluctuations of the bulk background fields consist of sums of functional integrals over both connected and disconnected world-sheets, rather than simply the connected world-sheets as in conventional (Neumann) open-string theory. The exponentiation of world-sheets plays an essential rôle in ensuring the cancellation of the novel Dirichlet divergences that arise when considering individual connected world-sheets [11, 12].

Scattering amplitudes in the background of a D-instanton exhibit point-like fixed angle behaviour order by order in perturbation theory around the instanton [12]. In fact this striking feature was one of the main reasons for the original interest in string theory with fully Dirichlet boundary conditions [10] (and contrasts with the exponential decrease of fixed-angle scattering cross sections in closed-string theory [33, 34]). Although

6

originally studied in the bosonic theory this point-like behaviour is also a feature of fixed-angle superstring amplitudes in perturbation theory in the background of a D-instanton [35]. A D-brane is not point-like – this is a simple consequence of the Neumann boundary condition in the time-like direction [35, 36, 37]. However, the length scale that characterizes its interactions order by order in perturbation theory, $\sqrt{\alpha'}$, does not take into account the excited modes of the brane. Since the effective tension of a $\mathrm{R} \otimes \mathrm{R}$ brane is of order $1/g\alpha'$ a new momentum scale should arise beyond which high frequency modes on the brane can be excited. The possible existence of such an intrinsically non-perturbative scale seems related to observations in [9].

Effective world-volume theories
The action of a D-instanton is particularly simple because all boundaries are mapped to the same point in the target space-time and (1) is simply an integral over the position of that point. With a single D-brane of $p \geq 0$ spatial dimensions the integration over y^i in (1) is (at first sight) replaced by a functional integration over the values of the world-volume fields $Y^i(x^\alpha), A_\alpha(x^\alpha)$ (and their fermionic partners) on the world-sheet boundaries. The resultant vacuum functional can be argued to have the form

$$Z = \int D^{9-p} Y^i(x^\alpha) D^{p+1} A_\beta(x^\alpha) e^{S^{(1)}[Y, A, \cdots]}, \tag{3}$$

where $S^{(1)}[Y, A, \cdots]$ is the $(p+1)$-dimensional world-volume action functional and the functional integral is over the fields living in this world-volume (the dots indicate the fermionic fields and dependence on the background fields is implicit). This action is itself the low-energy limit of a sum of conventional string functional integrals (over $X^\mu(z)$) on connected world-sheets with boundaries coupling to the massless open-string fields – the abelian Yang–Mills potential $A_\mu = (Y^i, A_\alpha)$ and its massless open-string fermionic partners – and on which the boundary conditions are Dirichlet in the directions labelled, i. Since the effective action originates from the ten-dimensional open string theory it possesses half the number of world-volume supersymmetries that the original type II theory has. This naturally agrees with the fact that a p-brane is a BPS soliton in the presence of which half the supersymmetry is spontaneously broken.

For example, in the case of a single D-string ($p = 1$) the effective action can be argued to have the Nambu–Goto/Born–Infeld form [29, 38, 39],

$$S^{(1)}[Y, A, \cdots] = \frac{-1}{2} \int d^2 x \left(e^{-\phi} \sqrt{-\det(G + \mathcal{F})} + \tfrac{1}{2} \epsilon^{\alpha\beta} B^R_{\alpha\beta} - \tfrac{1}{2} a \epsilon^{\alpha\beta} \mathcal{F}_{\alpha\beta} + \ldots + O(g^0) \right), \tag{4}$$

where the effective world-volume coordinates, x^α, are the zero modes of the original world-sheet fields and where $\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B^N_{\alpha\beta}$ and $G_{\alpha\beta}$ is the induced metric on the effective world-sheet. $F_{\alpha\beta} = \partial_{[\alpha} A_{\beta]}$ is the Maxwell field strength in the world volume and the dots indicate terms involving fermionic world-volume fields that complete the $N = 8$ supersymmetry of the world-sheet action (these fermions on the effective world-sheet

are components of the dimensionally reduced fermion of ten-dimensional supersymmetric Maxwell theory).

This effective Born–Infeld action may be motivated as in [38] by considering the modification of the string tree-level β function conditions in the presence of a constant open-string field strength – generalizing the discussion of the purely Neumann theory in[40, 41, 42]. In fact, the Born–Infeld action has long been known to arise from the open-string theory [43] and [44]. For fluctuations of the string around its straight configuration the induced metric is given by $G_{\alpha\beta} = \eta_{\alpha\beta} + \partial_\alpha Y^i \partial_\beta Y^i$ and the bosonic part of the Wess-Zumino term becomes $B_{\alpha\beta}^R + B_{ij}^R \partial_\alpha Y^i \partial_\beta Y^j$. The leading term in the exponent in (3) is again determined by the disk diagram while higher-order terms arise from connected world-sheets of higher genus. In [38] it was shown that the full dyonic spectrum of type IIB string solitons with tensions $T_{m,n} = \sqrt{(na - m)^2 + n^2/g^2}$ (where m, n label the NS $\otimes$ NS and R $\otimes$ R antisymmetric tensor charges, respectively) [16] can be summarized in the form (4) by considering n coincident D-strings with background $\mathcal{F}$. This is the spectrum expected from the arguments in [4], although the delicate issue of why the only stable states are those with m and n coprime requires a more subtle argument.

With n parallel D-branes there are n independent $U(1)$ gauge potentials associated with the ground states of open strings with both end-points on the same brane. These potentials may be denoted $A_\mu^{rr} = (A_\alpha^{rr}, A_i^{rr})$ where $r = 1, \cdots, n$ labels the brane (and $A_i^{rr} = Y_r^i - y_i^r$ defines its transverse position). There are also $n^2 - n$ gauge potentials associated with ground states of open strings with end-points on different branes, $A_\mu^{rs} = (A_\alpha)^{rs}, A_i^{rs})$ $(r \neq s)$. When the branes are separated the extra potentials are massive but they become massless in the limit of coincident branes – when all the branes are coincident there is a nonabelian $U(n)$ gauge symmetry [3, 4]. The dynamics of the system of parallel branes is thus related to spontaneously broken supersymmetric $U(n)$ Yang–Mills theory (with Yang–Mills potentials, A_α^{rs}) compactified from 10 dimensions to $p + 1$ dimensions where the scale of the breaking is determined by the separation of the branes. Very interestingly, since the spatial coordinates Y_i^r are also components of the ten-dimensional gauge potentials they also form part of a larger non-commutative algebra when the branes are closely spaced, as was emphasized in [4]. This is reminiscent of ideas in [45].

At second sight, however, (3) does not lead to a well-defined world-volume quantum theory since the theory with action $S^{(1)}$ is non-renormalizable. The world-volume theory should really be defined by the complete open superstring theory compactified to $p + 1$ dimensions, rather than by its low-energy approximation. In that case (3) would be an integral over open-string fields. This bears some resemblence to the suggestion, made for the case $p = 1$ in [6], that the world-volume manifold might be considered to be an approximation to the $(p+1)$-dimensional target space of a string theory. The case $p = 1$ is an example of the idea that a string world-sheet might be described by the target space of another string theory.

To summarize:-

- The classical world-volume theory of a $R \otimes R$ p-brane is obtained in the D-brane description as a low-energy limit of ten-dimensional open-string theory with Neumann boundary conditions in $p + 1$ directions and Dirichlet conditions in the remaining $9 - p$ directions. The ten-dimensional massless open-string ground state fields are interpreted as world-volume fields of the brane. Although the world-volume action may not be a well-defined p-brane quantum theory the full open-string theory is. The fact that the open-string loop expansion is a power series in g implies that the p-brane tension behaves like $1/g$ for small g.

The idea that p-branes are defined by an underlying string theory plausibly also applies to the p-branes of the $NS \otimes NS$ sector, such as the heterotic one-brane and five-brane:-

- The world-sheet action of the heterotic string (or 1-brane) may be obtained as the low-energy limit of the compactification of ten-dimensional type IIA theory on an orbifold, $K_3 \times T^4/Z_2 \times R^2$, where Z_2 acts on non-compact coordinates [5]. Furthermore, the $NS \otimes NS$ fivebrane soliton of the type IIA theory has a world-volume action that is the low-energy limit of the compactification of the type IIA theory on an orbifold, $T^4/Z_2 \times R^6$. Again, the underlying string theory gives a well-defined quantum theory even if the field theory is not. In these cases the target-space loop expansion is a power series in g^2 since the underlying theory is a closed-string theory so that the $NS \otimes NS$ p-brane tension behaves as $1/g^2$.

In both cases the classical world-volume fields are those that come from strings that are tied to the brane. In the case of D-branes these are the open-string fields while in the heterotic examples they are in the twisted closed-string sector. These are non-gravitational fields so the classical effective world-sheet theories are non-gravitational. However, in either case quantum loop corrections of the underlying world-volume string theory necessarily induce gravitons which move in the ten-dimensional target space so that the distinction between the world-volume and the target space will disappear in the quantum theory. This relationship between the world-volume theory and the underlying string theory can be summarized by the following table,

EFFECTIVE WORLD-VOLUME		UNDERLYING WORLD-SHEET
World-volume coordinates	x^α	World-sheet fields
World-volume fields	$A^i \equiv Y^i,\ A^\alpha$	Target-space fields
World-volume loop-counting parameter	$e^{\langle \phi \rangle}$	Target-space loop-counting parameter
$R \otimes R$ sector (tension)$^{-1}$	g	open string loop
$NS \otimes NS$ sector (tension)$^{-1}$	g^2	closed string loop

More generally, this 'confusion' between target space and world-volume is also illustrated by the profusion of rôles for the same effective action – it may describe a world-volume from one point of view or a target space from another. For example:-

- A two-brane soliton of eleven-dimensional M-theory can have a (string-like) boundary that lives in a five-brane soliton [39, 46, 47]. This self-dual string is described by a world-sheet embedded in the six-dimensional world-volume of the five-brane which itself is embedded in eleven space-time dimensions. The five-brane solution breaks half the eleven-dimensional supersymmetry and the embedded string soliton breaks half of the remaining supersymmetry. On the other hand the same self-dual string theory arises directly in six-dimensional space-time by the compactification of the type IIA theory from ten dimensions on K_3 [48, 49].

4 Boundary states, D-branes and space-time supersymmetry

The energy between two parallel D-branes is determined to lowest order by the cylinder diagram in which one boundary is fixed in the world-volume of one of the branes (at y_1^i, say) while the other is fixed in the other brane (at y_2^i). The expression for this diagram is given by

$$\sum_S \int d\tau \langle B^{(S)}, \eta_1, y_1 | e^{-H_S^{cl}\tau} P_{GSO} | B^{(S)}, \eta_2, y_2 \rangle, \tag{5}$$

where the sum is over the NS $\otimes$ NS and R $\otimes$ R closed-string sectors (labelled by S) and H_S^{cl} is the usual closed-string hamiltonian in the S sector. A sum over spin structures is implied by the GSO projection in the cylinder channel P_{GSO}. The end-states satisfy the world-sheet supersymmetry conditions [13, 3],

$$(F^{(S)} + i\eta\tilde{F}^{(S)})|B^{(S)}, \eta, y\rangle = 0, \tag{6}$$

where $F^{(S)}(\sigma)$, $\tilde{F}^{(S)}(\sigma)$ are the left-moving and right-moving world-sheet supercharges. The sign $\eta = \pm 1$ determines whether the state is a BPS state or an anti-BPS state. In the case that both end-states are of the same kind the sum over spin structures vanishes by Jacobi's abstruse identity which indicates that the energy vanishes due to a cancellation of the exchange of states in the R $\otimes$ R and the NS $\otimes$ NS sectors [13, 3].

More generally, the interaction of two BPS D-branes with M fundamental string states is determined by the cylinder diagram with vertex operators attached and is non-zero when $M \geq 2$. The arguments of [50, 13] (and references therein) show that in the case of D-instantons this process can be expressed in terms of position-space singularities on and inside the light-cone.

However, the correlation function of two boundary states of the opposite type – one BPS ($\eta_1 = 1$) and one anti-BPS ($\eta_2 = -1$) – does not vanish. This is due to a change in sign of one of the spin structures in the R $\otimes$ R sector (the one that is anti-symmetric along the axis of the cylinder), which breaks the supersymmetry completely. This is analogous to the fact that the BPS monopole - anti-monopole system is not a BPS state and not an exact solution to the equations of motion. The absence of supersymmetry in the correlation function of two boundary states of the opposite type is reflected in the D-instanton case by the presence of a singularity *outside* the light cone – at $(y_2 - y_1)^2 = \pi\alpha'/2$ – in this process, which makes the correlation function ill-defined [13]. Correspondingly, the correlation between two D-brane boundary states of opposite type is badly behaved at separations smaller than the string scale [51, 52], at $(\mathbf{y}_2 - \mathbf{y}_1)^2 = \pi\alpha'/2$ (where $\mathbf{y} = y^i$).

The fact that D-branes preserve half the space-time supersymmetry can be seen from a light-cone gauge argument that was originally applied to the case of D-instantons in [13] but which generalizes to the case of D-branes with $p \leq 7$ as in [14]. In the light-cone gauge the directions in the string world-sheet are identified with two of the target-space directions, $X^{\pm} = X^0 \pm X^9$. The time-like world-sheet coordinate is identified with the light-cone time, $X^+ = p^+\tau$, while the the other world-sheet coordinate, σ, is chosen so that $P^+(\sigma) = p^+/2\pi$. Now consider a process described by a world-sheet that is a semi-infinite cylinder spanned by an incoming physical closed-string state (at $X^+ = p^+\tau = -\infty$) evolving to a boundary end-state, $|B, y\rangle$ (the parameter η will be suppressed for now) at $X^+ = p^+\tau = y^+$. Since the boundary is at a fixed value of X^+ there is at least one direction in which the boundary condition is Dirichlet. Furthermore, in the light-cone gauge the coordinate $X^-(\sigma)$ is determined by $p^+\partial_\sigma X^- = \int_0^\sigma d\sigma' \partial_\tau X^I \partial_{\sigma'} X^I +$ fermion terms (where $I = 1, \ldots, 8$). It follows that $\partial_\sigma X^-|B\rangle = 0$ whether the transverse coordinates satisfy Neumann or Dirichlet boundary conditions so that X^- also satisfies a Dirichlet condition. This Dirichlet condition on X^- is consistent with the non-conservation of the conjugate momentum, p^+, at the boundary in the process under consideration. Thus, both X^+ and X^- satisfy Dirichlet conditions while the transverse coordinates X^I may satisfy either Neumann conditions, $\partial X^\alpha/\partial\tau|B, y\rangle = 0$ ($\alpha = 1, \ldots, p+1$), or Dirichlet conditions, $\partial X^i/\partial\sigma|B, y\rangle = 0$ ($i = p+1, \ldots, 8$). Therefore, the following argument will only apply to the case in which $p \leq 7$. Furthermore, in this situation the $p + 1$ Neumann directions are euclidean directions transverse to the world-sheet and the time direction is one of the $9 - p$ Dirichlet directions. This choice of coordinates does not describe a D-brane soliton but rather an analytic continuation in which time is one of the coordinates transverse to the $(p + 1)$-dimensional euclidean world-volume. Here, the p-brane is viewed as an instanton with action density localized in a region with $(p + 1)$-dimensional extension. For example, in the case $p = 5$ the euclidean theory in the space transverse to the five-brane can be interpreted as the euclidean continuation of the four-dimensional 'axionic' instanton, which has extension in the six dimensions (now taken to be euclidean) of the string world-volume (this interpretation has been used in the field theoretic con-

text in [53]). The total action will only be finite if the $p + 1$ Neumann dimensions are compactified on a torus.

A sixteen-component ten-dimensional supercharge decompose in the light-cone gauge into two inequivalent $SO(8)$ spinors, Q^a and $Q^{\dot{a}}$ ($a = 1, \cdots, 8$). In the type II theories there are two such supercharges. The left-moving supercharges satisfy the superalgebra

$$\{Q^a, Q^b\} = \delta^{ab}p^+, \quad \{Q^{\dot{a}}, Q^{\dot{b}}\} = \delta^{\dot{a}\dot{b}}H, \quad \{Q^a, Q^{\dot{a}}\} = \gamma^I_{a\dot{a}}p^I. \tag{7}$$

and a similar set of relations applies to the right-moving supercharges. In this expression p^I is the zero mode of the transverse momentum, P^I, and $H = P^-$ is the light-cone hamiltonian. The type IIA theory has left-moving and right-moving 16-component supercharges of the opposite chirality. In that case the role of the dotted and undotted $SO(8)$ spinors is interchanged between the left-moving and right-moving light-cone supercoordinates.

A Dirichlet boundary state that preserves half the space-time supersymmetry was constructed for the case of the purely Dirichlet theory in [13] and for general p in [14] – this should be viewed as a manifestly space-time supersymmetric version of the state in [3] that preserves half the world-sheet supersymmetry. The state is defined to satisfy the conditions,

$$(\partial + \tilde{\partial})X^i|B, \eta\rangle = 0, \qquad i = 1, \cdots, p + 1 \tag{8}$$

$$(\partial - \tilde{\partial})X^i|B, \eta\rangle = 0, \qquad i = p + 2, \cdots, 8, \tag{9}$$

on the bosonic coordinates and (in the case of the type IIB theory),

$$(S^a_n + i\eta M_{ab}\tilde{S}^b_{-n}) \mid B, \eta\rangle = 0, \tag{10}$$

on the fermionic coordinates (where S^a and $\tilde{S}^a$ are left-moving and right-moving light-cone gauge spin fields that are the fermionic target-space spinors of light-cone superstring theory). The matrix M_{ab} is given by

$$M_{ab} = (\gamma^1 \cdots \gamma^{p+1})_{ab}, \tag{11}$$

which reduces to $M_{ab} = \delta_{ab}$ for the case of the D-instanton and guarantees the linear relations between the supercharges,

$$(Q^a + i\eta M_{ab}\tilde{Q}^b) \mid B, \eta\rangle = 0, \qquad (Q^{\dot{a}} + i\eta M_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}}_{-n})|B, \eta\rangle = 0. \tag{12}$$

The type IIA case has a similar expression (in which the matrix M is the product of an odd number of γ matrices). These linear relations between the left-moving and right-moving space-time supercharges are consistent with the superalgebra and thus, one half of the space-time supersymmetry is preserved for any value of p, the case $p = -1$ (the D-instanton) coinciding with [13]. As expected, the conditions for $p > -1$ [54, 14] are

not Lorentz invariant since the boundary state is associated with the presence of a soliton background.

The supersymmetry conditions (12) can be expressed in light-cone superspace by introducing the $SO(8)$ spinor Grassmann coordinates, $\theta^a = Q^a - iM_{ab}\tilde{Q}^b$. The general boundary state is then a superfield that has a component expansion in powers of θ^a. Imposing the conditions (12) makes all components vanish apart from the bottom (for $\eta = 1$) or top (for $\eta = -1$) one. For example, for $p = -1$ the bottom component of the superfield is a complex combination of the two scalar fields, ϕ and a (the dilaton and the $R \otimes R$ scalar) and the top component is the complex conjugate. With $\eta = 1$ all components vanish apart from the bottom one and the boundary state can be written in the form,

$$|B\rangle = \exp \sum_{n=1}^{\infty} \left(\frac{-1}{n} \alpha_n^{i\dagger} \tilde{\alpha}_n^{i\dagger} + iS_n^{a\dagger} \tilde{S}_n^{a\dagger} \right) |B_0\rangle, \tag{13}$$

where the zero-mode piece is given by

$$|B_0\rangle = |i\rangle|\tilde{i}\rangle + i|\dot{a}\rangle|\tilde{\dot{a}}\rangle. \tag{14}$$

The states $|i\rangle$, $|\dot{a}\rangle$ are $SO(8)$ vector and spinor ground states in the left-moving sector and $\alpha_n^{i\dagger}$, $S_n^{a\dagger}$ are the creation modes of the bosonic and fermionic coordinates (and $\tilde{}$ denotes the right-moving sector). This boundary state couples equally to ϕ and ia and is therefore an equal source for the two scalar fields, which agrees with the field-theoretic treatment of the D-instanton in [22] (where $\alpha = ia$ is the $R \otimes R$ scalar in euclidean space). These statements generalize to arbitrary p, thereby reproducing (in linearized approximation) the relationships that give rise to the BPS states of the low energy field theory.

The basic p-branes are sources of $R \otimes R$ $(p + 1)$-form charges. More generally they are also sources of the $NS \otimes NS$ charges associated with B^N when there is a non-trivial boundary condensate of the open-string Maxwell field, A_α. Such branes are also sources of r-form $R \otimes R$ charges with $r = p - 1, p - 3, \cdots$ [4]. This is again a manifest consequence of the supersymmetric description of boundary states in the presence of open-string condensates [14] which resemble the convariant boundary states [54].

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